Asymptotic of one sum.

https://www.linkedin.com/feed/update/urn:li:activity:6636853075791491072

Let $(a_n)_{n\geq 1}, a_1 = 1, a_n = \sum_{k=2}^{n} \frac{k^2}{\sqrt[k]{k!}}, \forall n \geq 2.$ Calculate $\lim_{n\to\infty} \frac{a_n}{n^2}$.

Solution by Arkady Alt, **San Jose** ,**California**, **USA**.

Since $\lim_{n\to\infty} \frac{n}{\sqrt[n]{n!}} = e$ then $\lim_{n\to\infty} \frac{a_n - a_{n-1}}{n^2 - (n-1)^2} = \lim_{n\to\infty} \frac{\frac{n^2}{\sqrt[n]{n!}}}{2n-1} = \lim_{n\to\infty}\left(\frac{n}{2n-1} \cdot \frac{n}{\sqrt[n]{n!}}\right) = \frac{e}{2}$

and, therefore, by Stolz Theorem $\lim_{n\to\infty} \frac{a_n}{n^2} = \frac{e}{2}$.

Thus, $\sum_{k=2}^{n} \frac{k^2}{\sqrt[k]{k!}}$ asymptotically equivalent to $\frac{en^2}{2}$.